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Relational Understandings In Geometry: Implications From Study Of Interactions Between Knowledge Activation And Use

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Abstract

A principal requirement for students tackling plane geometry problems is to be able to work on the given figure(s) in ways that would help them get closer to the solution. In order to make satisfactory progress at this stage of the solution process, students need to access and use prior geometrical knowledge effectively. The effectiveness with which students are able to utilise their prior knowledge, largely depends on the quality of knowledge that they have developed as a consequence of learning experiences provided by teachers and other agents. In this paper, I draw upon a model of geometry knowledge development and highlight features of geometry knowledge that could assist students handle the early phase of diagram analysis that is critical for further progress in a problem-solving situation. The paper also provides suggestions for improving students' geometry knowledge base.

Introduction

The teaching and learning of geometry in our schools have undergone various changes over the years particularly with the advent of computers and research work on school learning. As a consequence of these developments, there is now a general trend towards helping students become more involved in the learning of and experimentation with geometry and related concepts. A major concern among teachers in encouraging students become active agents in the learning process has been to devise strategies that would help students visualize and interpret shapes better and recall their properties. While this aspect of learning of geometry is necessary, recent developments in learning mathematics and other school work (Anderson, 1990) are beginning to stress the need for students to understand the *relationships* between geometric figures, their properties and transformations performed on these figures. This shift in teaching strategy from one which is aimed at getting students to draw figures and memorise their properties to one which would help them recognise the links between the many geometrical figures is found in the goals of recent mathematics documents, such as the National Statement on Mathematics for Australian Schools (Australian Education Council, 1990). While these are welcome changes, teachers are seldom informed about why there is a need to understand relationships between the variety of geometrical forms and concepts that students encounter. In this paper, I draw on a model of geometry that was developed by Chinnappan and Lawson (1996) in order to

1. analyse the growth of students' understanding of the structure of geometric forms, and
2. suggest teaching and learning experiences that emphasise the establishment of connections between such forms.

How can we Visualise Geometric Knowledge Development?

Among the many models that have been advanced in the area of geometry learning and problem solving, the van Hiele's theory (Burger & Shaughnessy, 1986) about geometric thinking is well known to mathematics teachers. In this model, the levels proposed by van Hiele constitute an important development about patterns of reasoning that take place as students develop expertise with knowledge in the domain of geometry. However, the model does not provide a useful theoretical base from which to observe or document the development of geometric knowledge base. Specifically, the van Hiele model does not provide sufficient information on what it is that students reason about when they are dealing with geometry and related concepts. Following the current emphasis on helping students understand and construct meaningful relations with geometrical objects, there is a need for models that allow us as teachers to be able to make inferences about the quality of knowledge connections that students establish when they are working within a geometric environment.

In a series of studies about geometry problem solving, Chinnappan and Lawson (1996) investigated geometric componential knowledge that students accessed during the course of their solution attempt. These studies suggest that there is a link between the extent of network of knowledge associated with geometric concepts, and the use of these concepts during problem solving. On the basis of these studies, the authors advanced a model that attempted to provide a better picture about the nature of students' geometric knowledge base. In this model, the authors identified five levels at which students could exhibit structural relations between the various components of geometry taught in our classrooms: Basic Feature Level, Form Level, Rule Level, Application Level and Elaboration Level. For the purpose of this paper, I would like to focus on the first two of these levels, namely, Basic Feature and Form and illustrate how teachers could view student knowledge at these levels, and exploit it in ways that could help students. Let me outline the major aspects of these two levels.

Basic Feature Level knowledge

At this level the discrete features such as points, lines and curves, which constitute the building blocks of geometrical figures, are available for access when the student is questioned or provided with a diagram. These features are described as discrete in the sense that students are not required to establish relationship between the features.

Form Level knowledge

At the Form level, geometric forms and figures are accessed when the student is questioned or shown a diagram. These forms require the linking of knowledge items from Basic Feature level in order to form more complex geometrical representations such as squares and circles. A further development at this level is that a student is able to infer that a knowledge item at the Basic Feature level could take on different roles depending on where it is situated within a diagram. For example, although both the radius and tangent to a circle are straight lines, geometrically they have different properties and the two can interact in different ways. Radius is a straight line from the centre of a circle to the circumference while tangent is a straight line from a point outside a circle touching the circle. At this level, students' understandings must reflect these important knowledge connections.

Knowledge network at this level also involves connections between geometrical figures and forms, and relations between forms that are created by constructions carried out within a given figure. For example, given a square a student might choose to draw one of its diagonals. Having done this, the student might activate the knowledge that the diagonal bisects that square or go even further to suggest that it creates two congruent right-angled triangles.

Significantly, in the above analysis I have not referred to any knowledge of theorems, rules or formulae that teachers and students could associate with the knowledge at the above levels. This set of knowledge is analysed by Chinnappan and Lawson (1996) at the next level, namely Rules. In the remainder of this paper, I would like to provide further illustration of students' knowledge extension at the above two levels and consider their pedagogical implications.

Students' Analysis of Geometric Information at the Basic Feature and Form Levels

I chose to examine knowledge at these two levels for two reasons. Firstly, knowledge extension at the Basic Feature level mostly takes place as a result of the experiences provided in the primary mathematics curriculum. Secondly, some of the major problem-solving difficulties experienced by high school students could be traced to poor connections or extensions made at the Form level.

Let use examine some of the connections students could make at the levels of Basic Feature and Form through an example. In Figure 1, one could identify a number of Basic Features such as lines, circle and a triangle. Students need to process these bits of information differently in order to extend their knowledge from the Basic Feature level to Form level, a level at which one draws out the relational understanding that was addressed by Skemp (1971).

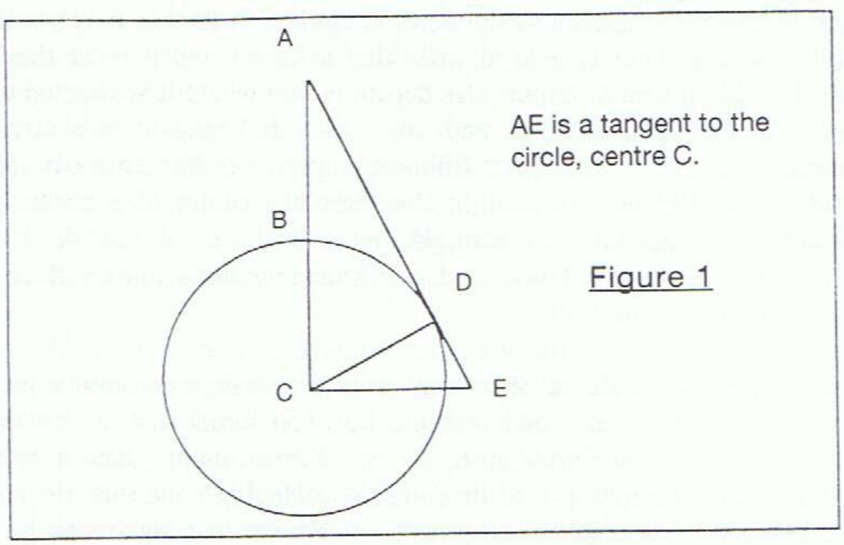

Figure 1

Students who have extended their knowledge at the Form level, should be able to go beyond the features and be able to recognise the structural functions of such basic features. By this I mean students should recognise that certain components can serve more than one role within a given diagram or a set of problem information. In Figure 1 for example, most students could recognise that line CD as a radius, but seldom do they consider the same line as the perpendicular height of the triangle ACE assuming these students could 'see' triangle ACE. Another example of this type of processing involves the radius CD again, but this time the student should be able to infer that CD is equal in length to CB as both are radii, and extend that connection to a point where they are able to recognise that CB is part of segment AC which is the hypotenuse of the right-angled triangle ACD. The establishment of the aforementioned relations is an active process, and requires students to go beyond knowledge about circles and triangles if they are to invoke theorems and formulae that could be associated with these newly constructed forms or relations which Greeno (1983) argued to underlie meaningful learning. Clearly, there is a need to integrate many bits of geometric knowledge, as inferences made about one part of the given figure is predicated on what was recognised in a related part of the figure.

Suggestions for Improving Students' Knowledge Extension at Form Level

A major reason for lack of depth in our students' knowledge at the Form level is that traditional teaching approaches do not emphasise such links or relations. The general approach to teaching geometric figures involves the construction of individual figures such as squares and rectangles on the basis of certain properties. Students learn these properties and associate them with the relevant figures. This set of information is important for the solution of a certain category of problems. However, problems that students encounter in tests and examinations in the later part of their geometry learning belong to a second category that calls for the activation and use of information that is more extensive, rich and better connected, the type of knowledge that students develop at the Form level. Knowledge of figures and their properties is necessary but not sufficient for making progress with novel problems that students would face in the more advanced stages of their secondary mathematics curriculum.

As teachers we need to take a more active role in helping students extend their knowledge at the Form level. Instead of relying on students to make the connections on the basis of knowledge about geometric figures, their properties and the solution of routine exercise problems, classroom experiences could shift the focus more to the construction and investigation of relations embedded composite figures such as Figure 1.

In order to facilitate the development of knowledge of the type outlined at the Form level, I suggest that teachers adopt the following approaches as part of their instruction:

- Students should be required to identify components in a given figure. For each of the components identified, students could be asked to explore and state the relations between the components. Students should also be encouraged to look for as many relations as possible for a set of components that was identified.

- If students missed out on the identification of key components, the teacher could intervene and provide with clues so that students attend to parts of the figure in which that component is located. The method of cueing has been shown to have beneficial effects in reinforcing and improving students' knowledge network in a range of subject areas.

- The above steps could be repeated but this time students could alter the orientation and location of the various components in the original figure.

- As a further activity, students could be asked to generate their own composite figures and carry out the componential and relational analysis outlined above.

The above suggestions can be carried out in group-based activities where members of the group could provide feedback on individual effort. Processing of the diagrams in this manner could prove to be a productive and enjoyable student activity. With the availability of many computer softwares that make drawing of figures less arduous and time consuming, teachers could devote more time to engendering *structural relations* between geometric figures. Such activities provide important avenues for students to improve and enrich the type of mathematical knowledge schemas that Sweller (1989) argued to have a powerful effect on problem categorisation and solution. Further, via such activities students are given the opportunity to acquire and practice more general problem-solving skills such as conjecturing and self reflection (Schoenfeld, 1985) which are equally important in helping students access prior geometric and related knowledge.

Conclusion

A major challenge for mathematics teachers is to identify and diagnose difficulties experienced by our students in their problem solving efforts. In order to achieve this task effectively, we need to understand

1. the complexities of the different phases involved in problem solving,
2. the nature of mathematical and other knowledge that would facilitate cognitive actions relevant to any particular phase in the solution process.

In this paper, I have addressed the quality of knowledge that would impact upon students' actions in analysing diagrams given or constructed by the student. This analysis suggests that the extent of knowledge network associated with understanding of structural relations between geometric forms could have a significant effect on the rest of the course of the solution process.

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